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Augmented Reality With Semarang Historical Buildings to Enhance Students' Geometry Modeling Skills

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Abstract

Mathematical modeling still being a challenging competency for many student in secondary school level, especially in geometry where abstract spatial conception still difficult to connect with real-world situations. This study examined changes in students' mathematical modeling skills following an AR-supported learning intervention integrated with Semarang City's historical buildings on students mathematical modeling skills in geometry. Mix methods with sequential explanatory design was used involving 32 students that selected through purposive sampling. Quantitative data were collected using pretest and posttest modeling tasks that based on Gereja Blenduk, Lawang Sewu, and Tugu Muda as historical heritage, then analyzed using the Shapiro-Wilk test, paired sample t-test, and N-Gain. Qualitative data was gained from classroom observation and informal questioning in learning proses were analyzed to explain the quantitative findings. The results of test showed a significant improvement in students modeling skills ($t(31) = 8.494, p < 0.001$) with a medium N-Gain of 0.5755. AR supported visualization, problem identification, and mathematization, but students still experienced difficulties in validating and interpreting mathematical results in real world contexts.

Introduction

Mathematical modeling is widely recognized as an essential competency in mathematics education, as it involves students in the process of extracting information from real-world situations, constructing mathematical models, solving those models, and interpreting results in context (Blum & Leiss, 2007; Suh et al., 2021). However, applying mathematical modeling in the domain of geometry presents challenges for students. Geometry topics require students to visualize abstract spatial relationships, recognize geometric structures in real objects, and mentally manipulate three-dimensional forms all before any mathematical model can be constructed (Battista, 2007; Medina Herrera et al., 2024). When these visualizations fail, students develop misconceptions, and their understanding of geometric concepts remains fragmented (Kusno & Sutarto, 2022). Furthermore, most geometry lessons in secondary school focus on formulas and procedures, providing students with little opportunity to engage in the full modeling process, including problem identification, mathematization, model validation, and interpretation (Cevikbas et al., 2022; Schukajlow et al., 2018). Students rarely experience transitions between the real-world context and the mathematical domain, which limits the development of genuine modeling competence (Yuliati & Lestari, 2019).

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Visualization lies at the heart of geometry-based mathematical modeling. Before students can formulate a mathematical model, they must first observe a real object, recognize its geometric properties, identify relevant measurements, and translate those observations into mathematical language (Battista, 2007; Medina Herrera et al., 2024). Textbook images and static diagrams are limited in this regard because they cannot convey spatial depth, proportions, or the three-dimensional relationships that students need to perceive (Žakelj & Klančar, 2022). Without adequate visual support, students struggle to bridge the gap between real-world contexts and geometric abstraction, which is one of the key reasons why the representation and formulation stages of the modeling cycle remain particularly difficult (Cevikbas et al., 2022; Kaiser & Brand, 2015). A learning tool that allows students to interactively explore geometric structures from multiple angles could therefore play a meaningful role in supporting the early and middle stages of the modeling process (Greefrath, 2011).

Augmented Reality (AR) is one such technology. AR overlays digital content, such as three-dimensional models and spatial annotations, onto the real physical environment in real time, allowing students to interact with virtual geometric objects while remaining in their actual surroundings (Ibáñez & Delgado-Kloos, 2018). Unlike virtual reality, AR maintains a connection to the physical world, making it particularly suitable for contextual geometry learning. Research has shown that AR supports spatial visualization, conceptual understanding, and student engagement in geometry tasks (Del Cerro Velázquez & Méndez, 2021; Garzón & Acevedo, 2019). Demitriadou et al. (2020) found that AR consistently produced better geometry learning outcomes than conventional approaches. Greefrath (2011), extending Blum & Leiss (2007), argued that digital tools can support each stage of the modeling cycle, from understanding the problem to verifying results. However, Cevikbas et al. (2023) cautioned that the effectiveness of digital tools in modeling education depends on how well the pedagogical design aligns with the cognitive demands of each modeling stage, and that providing access to tools alone is insufficient without purposeful instructional integration (Maritsa et al., 2021).

The choice of learning context also matters for geometry modeling. Contextually meaningful problems increase student engagement and support the problem identification stage of the modeling cycle, as students can draw on prior knowledge and cultural familiarity when encountering the problem (Kohen & Orenstein, 2021). Historical buildings offer a rich source of authentic geometric structures that are rarely encountered in textbooks. Cahyono (2018) and Cahyono & Ludwig (2019) demonstrated that learning experiences embedded in local heritage environments can effectively support mathematical literacy and modeling. Semarang City's historical buildings, including Gereja Blenduk, Lawang Sewu, and Tugu Muda, contain a variety of circular and cylindrical geometric forms that are directly relevant to secondary school geometry. These buildings are also culturally familiar to students in Semarang, making them appropriate as authentic and meaningful real-world problem contexts that connect mathematics to students' own environments (Izzatin & Dewi, 2025; Solihin et al., 2025; Rais, 2025).

Despite the potential of AR in geometry education, few studies have explicitly mapped AR use to each specific stage of the mathematical modeling cycle, particularly in geometry contexts grounded in local cultural heritage. Most existing AR studies in mathematics education rely on generic everyday objects as contexts, without connecting AR affordances to the specific cognitive demands of each modeling stage a gap identified by Krawitz et al. (2025) and Greefrath & Siller (2024). This study addresses that gap by integrating AR with historically significant buildings in Semarang City and explicitly aligning each AR feature with a specific stage of the modeling cycle. The novelty of this study lies in this deliberate alignment, which aims to support students not only in the early stages of visualization and problem identification but also in the more advanced stages of model formulation and validation. The findings are

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expected to benefit students by supporting the development of mathematical modeling skills, provide practical guidance for teachers designing technology-integrated geometry lessons, and serve as a reference for curriculum developers exploring culturally responsive digital learning.

Method

Research Design

This study employed a quantitatively driven explanatory mixed-methods design to examine changes in students' mathematical modeling skills and to explain students' learning experiences during augmented reality-supported geometry learning. The quantitative component served as the primary strand and used a one-group pretest–posttest design. Students' mathematical modeling skills were measured before and after they participated in the instructional intervention. The qualitative component consisted of structured classroom observations, informal questioning, field notes, students' written responses, and classroom reflection records. The qualitative evidence was used to explain the patterns identified in the quantitative findings, particularly the differences in improvement across the five mathematical modeling indicators.

The one-group pretest–posttest design was selected because the study was conducted in a natural classroom setting in which random assignment and the establishment of a control group were not feasible. Therefore, the findings were interpreted as evidence of changes in the participating students' mathematical modeling skills following the intervention. The results were not interpreted as definitive evidence that the AR-supported intervention was the sole cause of the observed changes because possible influences such as repeated testing, maturation, ordinary classroom learning, and other contextual factors could not be completely controlled.

Research Setting, Participants, and Sampling

The study was conducted at a secondary school in Semarang City, Indonesia. The participants were 32 Grade VIII students aged between 13 and 14 years, consisting of 19 male and 13 female students. Based on the school's academic records, the class contained students with low, medium, and high levels of academic achievement. Before the intervention, the classroom teacher confirmed that all participating students had completed the Grade VII geometry curriculum, including the basic properties of circles. However, they had not previously participated in mathematics learning involving augmented reality or contextual mathematical modeling activities.

Purposive sampling was applied at the school, class, and student levels. The school was selected because it was accessible to the researchers, the mathematics teacher was willing to participate, and sufficient mobile devices were available to support group-based AR learning. The school had also not previously implemented AR-supported mathematics instruction. The class was selected because it represented mixed academic achievement levels and had completed the prerequisite geometry content.

Students were included when they were officially enrolled in the selected class, had completed the required circle-geometry content, had no previous experience with AR-supported mathematics learning, and attended the principal stages of data collection. The purposive selection of one class from one school limits the external validity of the findings. Consequently, the results should be interpreted as context-specific evidence and should not be generalized automatically to broader student populations.

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AR-Supported Learning Environment

The instructional intervention integrated augmented reality with contextual geometry problems based on three historical buildings in Semarang City: Tugu Muda, Gereja Blenduk, and Lawang Sewu. These buildings were selected because they contain visible circular, curved, arched, and cylindrical structures that are relevant to Grade VIII circle geometry. The buildings are also culturally familiar to students in Semarang, making them meaningful real-world contexts through which formal geometric concepts could be connected with students' surrounding environment.

The AR learning environment, referred to as BangunSemar, contained three-dimensional representations of the selected historical buildings. The three-dimensional models were constructed using Tinkercad based on reference photographs and the geometric features required in the learning tasks. The models were subsequently processed using Assemblr Studio to generate mobile AR content and AR markers. ThingLink was used to organize access to the instructional content and digital learning resources.

Students accessed the AR models by scanning markers using mobile devices. They could rotate, enlarge, reduce, and observe the three-dimensional models from different viewpoints. The AR models highlighted geometric features such as circular gardens, domes, arches, curved roofs, angles, radius, diameter, and dimensional information. These functions were intended to help students recognize mathematical information that might be difficult to identify through static textbook diagrams.

Instructional Framework

The intervention was guided by Greefrath's technology-supported mathematical modeling framework, which extends the mathematical modeling cycle proposed by Blum and Leiss. In the present study, the modeling process was operationalized through five indicators: identifying and understanding the problem, constructing mathematical representations, formulating and solving mathematical models, validating mathematical results, and interpreting solutions in the original context.

Each AR feature was deliberately aligned with a particular stage of the modeling cycle. Rotation, zoom, and multi-angle viewing supported students in observing historical buildings, identifying geometric elements, and constructing mathematical representations. Dimensional information and highlighted geometric features supported students in determining relevant variables, selecting formulas, and formulating mathematical models. Comparisons between students' calculated results and the dimensions or appearance of the AR models were used to encourage validation. Reflective questions were also used to guide students in explaining the contextual meaning of their numerical results.

This alignment ensured that AR functioned as an instructional scaffold rather than merely as an attractive visual supplement. The learning design particularly emphasized students' movement from the real-world context of the historical buildings to the mathematical domain and their return from mathematical results to contextual interpretation.

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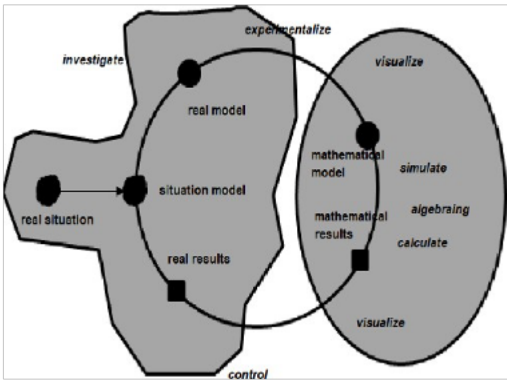

Figure 1. Modified Mathematical Modeling Cycle Greefrath (2011) based on Blum & Leiss (2007)

Instructional Intervention Procedure

The intervention was conducted over four meetings, with each meeting lasting 90 minutes. The meetings were completed within two weeks and consisted of a pretest session, two AR-supported instructional sessions, and a posttest and reflection session. Due to school regulations restricting individual mobile-device use, students worked in groups and shared one mobile device within each group.

During the AR-supported sessions, group members naturally assumed different roles. Some students operated the AR application and scanned the markers, while others recorded geometric information, constructed representations, selected formulas, performed calculations, or wrote the group's final response. The classroom teacher facilitated the learning process by clarifying instructions, monitoring group activities, and prompting students to explain their reasoning. The researcher observed students' modeling processes, completed the observation sheet, and asked non-leading questions when clarification of students' reasoning was needed.

The sequence of learning activities, instructional objectives, use of AR, and expected mathematical modeling outputs is presented in Table 1.

Table 1. Learning Activities during the Intervention

Meeting	Duration	Learning Objectives	Activities and AR Use	Expected Modeling Output
1	90 minutes	To measure students' initial mathematical modeling skills	Students completed an individual pretest consisting of three open-ended contextual problems. AR was not used during the pretest. The researcher explained the subsequent learning procedures after the test.	Baseline scores for the five mathematical modeling indicators
2	90 minutes	To apply circumference and cost estimation using Tugu Muda and central and inscribed angles	Students worked in groups and scanned the AR markers. They observed the models from different viewpoints, identified geometric	A circumference and fencing-cost model based on Tugu Muda and an inscribed-angle

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		using Gereja Blenduk	features, recorded known and required information, constructed sketches, selected formulas, and completed calculations. The teacher facilitated the activities. The two topics were combined because of school scheduling constraints.	model based on Gereja Blenduk
3	90 minutes	To calculate arc length and sector area using Lawang Sewu and apply the complete modeling cycle	Students observed the Lawang Sewu AR model, identified arc parameters, represented the curved structure as part of a circle, formulated and solved the model, checked the result against the AR model, and interpreted the result in the building context. The teacher provided reflective prompts.	An arclength and sector-area model accompanied by validation and practical interpretation
4	90 minutes	To measure students' post-intervention skills and consolidate their learning experiences	Students completed the posttest individually using the same instrument as the pretest. AR was not used. After the test, students participated in a classroom reflection concerning the usefulness and challenges of AR-supported learning.	Posttest scores for the five modeling indicators and qualitative reflection records

During the first meeting, students completed the mathematical modeling pretest individually without access to AR. The test was used to establish their initial mathematical modeling performance across the five indicators. During the second meeting, students examined the circular garden surrounding Tugu Muda and calculated its circumference and fencing cost. They also examined the dome-related features of Gereja Blenduk to determine the relationship between central and inscribed angles.

The third meeting used the Lawang Sewu context to engage students in the complete mathematical modeling cycle. Students identified the radius, central angle, and other relevant information from the AR model. They represented the curved structure as a sector or part of a circle, formulated equations for arc length and sector area, completed the calculations, checked whether their results were reasonable, and explained what the results represented in the building context.

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During the fourth meeting, students completed the posttest individually without using AR. The same test was administered at the pretest and posttest stages to provide direct comparability. However, using the same instrument may have introduced a practice effect because students had previously encountered the problem contexts. This possibility was considered limited because the tasks required multistep mathematical reasoning, written justification, validation, and contextual interpretation rather than simple recall.

Research Instruments

The study used a mathematical modeling skills test, a structured classroom observation sheet, an informal questioning guide, field notes, students' written responses, and classroom reflection records. The mathematical modeling test provided the primary quantitative data, whereas observations, informal questioning, written responses, and reflections provided qualitative data used to explain the quantitative findings.

In addition to the data-collection instruments, the study used a Learning Objective Sequence or *Alur Tujuan Pembelajaran*, a teaching module, and AR-supported learning media. These instructional materials were developed in accordance with the Grade VIII geometry curriculum, the instructional objectives, and the mathematical modeling framework adopted in the study.

Mathematical Modeling Skills Test

The mathematical modeling skills test consisted of three open-ended contextual geometry problems based on Tugu Muda, Gereja Blenduk, and Lawang Sewu. The problems were designed to assess students' ability to transform a real-world situation into a mathematical representation, formulate and solve a mathematical model, validate the result, and interpret the solution in its original context.

The first problem was based on the circular garden surrounding Tugu Muda. Students were asked to identify the relevant dimensions, determine the radius when necessary, calculate the circumference, and estimate the total cost of installing a fence. The second problem was based on Gereja Blenduk. Students were required to determine an inscribed angle from a given central angle and explain the relationship between the two angles within the context of painting or decorating part of the dome. The third problem was based on a curved roof or arch at Lawang Sewu. Students were asked to represent the curved structure as part of a circle, calculate its arc length and sector area, and explain the practical meaning of the results.

Students' responses were evaluated according to five mathematical modeling indicators. Problem identification assessed students' ability to identify known and required information. Representation assessed their ability to construct an appropriate drawing, diagram, symbol, or geometric model. Model formulation assessed their ability to select formulas, establish mathematical relationships, and perform calculations. Validation assessed their ability to check whether a mathematical result was accurate and reasonable. Interpretation assessed their ability to explain the result meaningfully within the historical-building context.

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Table 2. Mathematical Modeling Skills Scoring Rubric

Score	Problem Identification	Representation	Model Formulation	Validation	Interpretation
0	No answer or relevant information is provided	No drawing or mathematical representation is provided	No formula or calculation is provided	No checking process is provided	No interpretation is provided
1	Known and required information is identified incorrectly	The drawing or representation is incorrect or irrelevant	The formula and calculation are incorrect	The checking process is incorrect	The interpretation of the result is incorrect
2	The problem is identified partially correctly, but relevant information is incomplete	The representation is partially correct or incomplete	The formula or calculation is partially correct	The checking process is partially correct but incomplete	The interpretation is partially correct but does not fully explain the contextual meaning
3	Known and required information is identified completely and correctly	A complete and mathematically appropriate representation is constructed	An appropriate formula is selected and the calculation is accurate	The result is validated systematically and correctly	The result is interpreted meaningfully and appropriately within the original context

Each indicator was scored from 0 to 3. A score of 0 indicated no relevant evidence, a score of 1 indicated an incorrect or minimally developed response, a score of 2 indicated a partially correct but incomplete response, and a score of 3 indicated a complete and accurate response.

Scores from the three problems were synthesized for each modeling indicator. Each indicator contributed a maximum composite score of 3, producing a maximum total test score of 15. This scoring system enabled the researchers to examine both overall mathematical modeling performance and indicator-specific development.

Classroom Observation and Informal Questioning

A structured observation sheet was used during the second and third meetings to document students' engagement with the mathematical modeling process. The observation focused on how students identified important information from the AR context, transformed building structures into geometric representations, selected and used appropriate formulas, checked their calculations, and interpreted their results in the original building context.

The observation protocol also documented students' group interactions, participation patterns, use of mobile devices, task progression, and distribution of roles. These aspects were recorded because students shared mobile devices and completed the AR-supported activities collaboratively.

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Informal questioning was conducted while students were working on the learning tasks. The researcher asked open-ended and non-leading questions to capture students' reasoning at the moment they made mathematical decisions. Examples included, "What information did you obtain from the AR model?", "Which geometric shape represents this part of the building?", "Why did you select this formula?", "Does your result make sense for the actual building?", and "What does this numerical result mean in the problem context?"

Students' verbal responses were summarized in field notes together with the meeting, task, participant code, and corresponding mathematical modeling stage. The informal questions were used to clarify students' reasoning and were not intended to guide them toward a particular answer.

Instrument Validation

Before implementation, the Learning Objective Sequence, teaching module, AR-supported learning media, mathematical modeling test, and scoring rubric were reviewed by two validators. The validators consisted of a mathematics education expert and an experienced mathematics learning practitioner. Each instrument was evaluated using a structured validation sheet tailored to its purpose.

The Learning Objective Sequence and teaching module were reviewed based on alignment with the Merdeka Curriculum, coherence of learning objectives, appropriateness of the content for Grade VIII students, suitability of the allocated learning time, and integration of the *Profil Pelajar Pancasila*. The AR-supported media were reviewed in terms of content accuracy, visual design, usability, navigation, readability, technical functionality, relevance of the three-dimensional models, and instructional innovation.

The mathematical modeling test was evaluated using a five-point rating scale covering content relevance, alignment with the five modeling indicators, accuracy of geometric concepts, appropriateness of the historical-building contexts, item construction, clarity of instructions, visual presentation, and language. The scoring rubric was also reviewed to determine whether the performance levels were clear and distinguishable.

Both validators approved the instruments with minor revisions. The primary recommendation concerned the clarity of several illustrations and geometric figures in the test items. The illustrations were revised so that students could interpret the building contexts and dimensions more accurately. All recommendations were addressed before the instruments were administered.

Scoring Consistency

All pretest and posttest responses were scored by the researcher using the same analytic rubric. Each student response was examined for evidence of problem identification, representation, model formulation, validation, and interpretation.

To examine intra-rater consistency, ten answer sheets were selected randomly and rescored two weeks after the initial scoring. The researcher did not refer to the original scores during rescoring. No discrepancies were found between the first and second scoring results, indicating stable application of the rubric over time. This procedure demonstrated intra-rater consistency, although it did not establish inter-rater reliability because the final scoring was completed by one researcher.

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Data Collection Procedure

Data collection followed the sequence of the four instructional meetings. Before the intervention, students completed the pretest individually under controlled classroom conditions. During the second and third meetings, the researcher recorded observation-sheet entries and field notes while students completed the AR-supported learning tasks. Informal questions were directed to students or groups when clarification of their reasoning was needed.

At the end of the intervention, students completed the posttest individually under conditions comparable to those of the pretest. The pretest and posttest answer sheets constituted the primary quantitative dataset. Observation records, field notes, informal-questioning responses, selected written responses, and classroom reflection records constituted the qualitative dataset.

Students were identified using codes such as S1, S2, and S3, whereas teachers were identified using codes such as T1 and T2. These codes were used in the analysis and reporting process to protect participant confidentiality.

Quantitative Data Analysis

Quantitative analysis began with descriptive statistics for the pretest and posttest scores, including the mean, standard deviation, minimum score, maximum score, and number of participants. These statistics were used to describe students' mathematical modeling performance before and after the intervention.

The normality assumption was examined using the Shapiro–Wilk test applied to the paired difference scores. The difference score for each student was calculated by subtracting the pretest score from the corresponding posttest score. When the normality assumption was satisfied, a paired-samples *t*-test was conducted at a significance level of $\alpha = 0.05$.

The paired-samples *t*-test results were reported using the mean difference, standard deviation of the difference scores, *t* statistic, degrees of freedom, *p* value, and 95% confidence interval. A *p* value below 0.05 was interpreted as evidence of a statistically significant difference between the pretest and posttest scores.

Cohen's (d_z) was calculated to estimate the magnitude of the paired difference. The effect size was calculated by dividing the mean paired difference by the standard deviation of the paired difference scores. The effect size was interpreted together with the significance test and confidence interval. Individual N-Gain scores were calculated to determine the number and percentage of students in each category. Indicator-specific N-Gain values were also calculated for problem identification, representation, model formulation, validation, and interpretation.

Because the study did not include a control group, the paired-samples *t*-test, effect size, confidence interval, and N-Gain were interpreted as complementary descriptions of change within the participating class rather than definitive causal estimates.

Qualitative Data Analysis

Qualitative data were analyzed thematically to explain the quantitative results, particularly the differences in improvement among the five mathematical modeling indicators. The analysis involved data familiarization, initial coding, comparison of codes, theme development, triangulation, and selection of representative evidence.

During familiarization, the researcher repeatedly reviewed the observation sheets, field notes, informal-questioning records, classroom reflection notes, and selected written responses. The

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records were organized according to meeting, task, participant code, and mathematical modeling indicator.

Initial coding was primarily deductive because the five mathematical modeling indicators served as the main analytical framework. Statements and observed actions were initially classified as evidence of problem identification, representation, model formulation, validation, or interpretation. Inductive subcodes were then developed to represent recurring classroom behaviors.

Table 3. Qualitative Themes from Classroom Observation and Informal Questioning

Theme	**Mathematical Modeling Indicator(s)**	**Principal Evidence**
AR-supported visualization of geometric structures	Problem Identification	Students identified circular shapes such as domes, arches, windows, and gardens more readily through AR than through static textbook diagrams
Improved identification of geometric information	Problem Identification	Students used the AR models to locate radius, diameter, angles, and other relevant information and connected these elements with appropriate formulas
Bypassing geometric representation	Representation	Many students proceeded directly to formula selection and calculation without drawing or labeling an explicit geometric representation
Easier mathematization through the AR context	Model Formulation	AR helped students move from the real-world building context to mathematical formulas and calculations with greater confidence
Continuing difficulty with validation and interpretation	Validation and Interpretation	Students often completed calculations but did not check the reasonableness of their answers or explain the results in the original building context

Within these themes, the researcher developed subcodes such as recognizing circular structures through AR, identifying radius and diameter, linking building features to circle formulas, moving directly from visualization to calculation, omitting geometric sketches, selecting formulas confidently, failing to return to the AR model after calculation, ending answers with numerical results, and omitting contextual explanations.

Students' verbal responses were compared with observation records and written answers. A theme was retained when it recurred across students or groups, was supported by more than one source of evidence, and contributed to explaining a quantitative finding. Preliminary codes and themes were also discussed with a peer to improve the coherence and credibility of the qualitative interpretation.

Representative excerpts were selected because they clearly illustrated recurring patterns. Statements originally expressed in Indonesian were translated into English for publication. Where necessary, the original Indonesian statements could be presented alongside the English translation to preserve the intended meaning.

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Integration of Quantitative and Qualitative Findings

Quantitative and qualitative findings were integrated during the interpretation stage. The quantitative analysis first established the overall pretest–posttest difference, effect size, confidence interval, N-Gain category, and improvement in each mathematical modeling indicator. The qualitative themes were then used to explain why particular indicators showed stronger or weaker development.

For example, students’ use of rotation and zoom functions to recognize circular structures helped explain improvement in problem identification. Their ability to locate the radius, diameter, and angle information supported the development of model formulation. In contrast, observation and written-response evidence showing that students frequently omitted sketches helped explain the relatively limited improvement in representation.

Similarly, students’ tendency to end their answers after obtaining a numerical result, without checking or interpreting it, helped explain the weaker development of validation and interpretation. In this way, qualitative evidence was used to produce an explanatory account of the quantitative patterns rather than being presented merely as a separate collection of quotations.

Result and Discussion

Quantitative Results

The study began by administering a pretest to measure students' initial mathematical modeling skills before the AR-supported intervention. After four meetings of AR-supported learning, a posttest was administered using the same instrument. Table 4 presents the descriptive statistics of both assessments.

Table 4. Descriptive Statistics of Pretest and Posttest Scores

Statistic	Pretest	Posttest
Mean	3.47	10.09
Standard Deviation	3.776	2.131
Number of Students	32	32
Minimum	0	4
Maximum	11	13

The mean pretest score of 3.47 indicates that students had limited prior ability in mathematical modeling tasks derived from contextual building problems. The low pretest scores, with several students scoring zero, reflect students limited prior exposure to contextual mathematical modeling tasks rather than an absence of basic geometry knowledge, a condition confirmed by teachers prior to the intervention. After the intervention, the mean posttest score increased to 10.09, suggesting a substantial improvement following the AR-supported learning sessions.

An examination of pretest answer sheets revealed patterns illustrating the nature of students’ initial difficulties. Many students were unable to correctly identify the mathematical information needed from the problem context. In Problem 1 (Tugu Muda), several students left the problem identification step blank or wrote only the diameter without identifying the radius or the required formula. In Problem 3 (Lawang Sewu), most students could not construct any

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geometric representation or formulate a model; several students wrote only the problem number and left the answer space empty. When probed informally before the intervention began, students indicated that they found it difficult to visualize what the building context looked like mathematically and did not know where to begin. These pretest patterns confirm that initial modeling difficulties were primarily situated at the problem identification and representation stages precisely the stages where AR-supported learning showed the highest N-Gain improvement in the posttest (Problem Identification $g = 0.94$, Model Formulation $g = 0.73$). Prior to hypothesis testing, a normality test was conducted on the difference scores using the Shapiro-Wilk test. Table 5 presents the results.

Table 5. Normality Test Results (Shapiro-Wilk)

Variable	**Statistic**	**df**	**Sig.**
Difference	.947	32	.122

The difference scores were normally distributed ($p = 0.122 > 0.05$), satisfying the assumption required for the paired sample t-test. A paired sample t-test was then conducted to examine whether the improvement was statistically significant. Table 6 presents the results.

Table 6. Paired Sample T-Test Results

Comparison	**Mean Difference**	**Standard Deviation**	***t***	**df**	**Sig.**
Posttest–Pretest	6.625	4.412	8.494	31	< 0.001

The results confirmed a statistically significant difference between pretest and posttest scores ($t(31) = 8.494$, $p < 0.001$), indicating that students' mathematical modeling skills improved significantly following the AR-supported intervention. To determine the magnitude of improvement, a normalized gain score (N-Gain) was calculated for each student. Table 7 presents the distribution of N-Gain categories.

To further characterize the magnitude of improvement, Cohen's d was calculated as the effect size for the paired sample t-test. Using the mean difference of 6.625 and the standard deviation of the difference scores of 4.412, Cohen's $d = 6.625 / 4.412 = 1.50$, indicating a large effect size. The 95% confidence interval for the mean difference was [5.034, 8.216], suggesting that the true mean improvement in the population is likely between approximately 5.0 and 8.2 score points. The minimum pretest score was 0 and the maximum was 11, while the minimum posttest score was 4 and the maximum was 13, reflecting a general upward shift in score distribution following the intervention.

Table 7. N-Gain Category Distribution

Category	**N**	**Percentage**
High ((g > 0.7))	9	28.125%
Medium ((0.3 \leq g \leq 0.7))	17	53.125%
Low ((g < 0.3))	6	18.75%
Mean N-Gain	0.5755	Medium

The average N-Gain of 0.57 falls into the medium category ($0.3 \leq g \leq 0.7$), indicate a meaningful but moderate level of improvement. Most students (53.13%) achieved medium improvement, while 9 students (28.13%) achieved high improvement. Six students (18.75%) remained in the low category, including those who had relatively higher pretest scores with minimal further improvement.

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To provide a more detailed picture of students' improvement, scores were also analyzed per modeling indicator. Table 8 presents the mean pretest score, posttest score, and N-Gain for each of the five indicators across all students and all three problems.

Table 8. N-Gain score per Modeling Indicator

Indicator	N-Gain	Category
Problem Identification	0.9394	High
Representation	0.2500	Low
Model Formulation	0.7286	High
Validation	0.4583	Medium
Interpretation	0.5013	Medium

Table 8 shows that the highest N-Gain was achieved in Problem Identification (g = 0.94, high category), followed by Model Formulation (g = 0.72, high), Validation (g = 0.45, medium), and Interpretation (g = 0.50). In contrast, Representation (g = 0.25, low) showed the smallest gains, indicate that most students remained in the "partially correct" range for the indicators even after the intervention.

Qualitative Findings

To support and explain the quantitative findings, qualitative data were collected through informal questioning and classroom observation during the AR learning sessions. Before the intervention, teachers described the initial condition of students' mathematical abilities. Teacher 1 (T1) stated,

> *"Honestly, they are still not very capable. The students can memorize formulas when taught, but when I ask them to observe objects around them or give them word problems, they become confused. Their mathematical literacy and mathematical modeling skills are still lacking."*

T2 added,

> *"They are accustomed to solving basic exercises and performing routine calculations. However, when faced with contextual problems that require deeper thinking, they immediately struggle."*

These statements confirm that students initially had low mathematical modeling skills, particularly in contextual problems, which is consistent with the low pretest scores shown in Table 4.

Indicator 1 Problem Identification and Indicator 2 Representation

During the AR learning process, when asked about the geometric shapes they could identify in the historical buildings, most students were able to recognize circular elements such as windows, arches, and domes. Student 1 (S1) said,

> *"The dome looks like a circle."*

S2 added,

> *"From above, it looks like a circle."*

Observations showed that students who previously struggled with textbook diagrams could identify circular shapes more readily using AR. This finding helps explain the improvement observed in the problem identification and representation indicators in the posttest.

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When asked what information they needed to find the area or circumference of a circular element, students could identify relevant measurements by referring to the AR model. S3 explained,

> *"The garden is circular, so we need to find the radius to calculate the circumference."*

S4 responded, to find the total cost of the fence, we first need to calculate the circumference and then multiply it by the cost per meter. Observations revealed that students spent less time clarifying the problem and more time discussing mathematical strategies compared to conventional lessons. This supports the improvement seen in the mathematization and formulation indicators. Although many students were able to identify geometric shapes from the AR models, they often did not translate this understanding into written geometric representations. Observation notes showed that students tended to move directly to calculations after selecting a formula, without first drawing or sketching the relevant geometric model. For example, in Problem 1 (Tugu Muda), most students applied the circumference formula immediately without drawing the circular garden and labeling its dimensions. A similar pattern was found in Problem 3 (Lawang Sewu), where only a few students sketched the sector before calculating arc length or sector area. When asked why they did not draw the shapes, several students stated that they considered drawing unnecessary once they had identified the correct formula. This suggests that students viewed geometric representation as a supporting rather than essential part of the modeling process, which may explain the relatively low N-Gain for this indicator (g = 0.25).

Indicator 3 Model Formulation

When asked about problems involving central and inscribed angles, students could relate the angles shown in the AR model to circle formulas. S5 explained,

> *"Based on your explanation about central and inscribed angles, for this problem we just need to find the inscribed angle, which is half of the central angle."*

S6 responded,

> *"When working on regular problems, I often get confused, but with AR it's easier to identify the given information and what needs to be found."*

These responses indicate that AR helped students move from the real-world context to the mathematical domain more effectively.

Indicator 4 Validation and Indicator 5 Interpretation

One theme emerged here: (4) difficulty in validation and interpretation. Despite successful calculation, most students did not return to the real-world context to verify or interpret results indicating AR visualization alone did not scaffold the metacognitive demands of these final stages. This theme supports the medium N-Gain for Validation (g = 0.46) and Interpretation (g = 0.50) and is developed theoretically in the Discussion. Based on observation of students' written work, most students focused primarily on applying circle formulas and performing calculations correctly, but some of their answers did not include any interpretation or validation of whether the results made sense in the actual building context. Students wrote final numerical answers without connecting them back to the original problem situation. For example, in Problem 3 (Lawang Sewu), S7 correctly calculated the arc length as 22 meters and the sector area as 154 m^2 but did not explain how these results related to the practical context of the arch frame length or the area to be painted. Similarly, in Problem 1 (Tugu Muda), S8 correctly calculated the circumference as 62.8 meters but did not check whether the result was reasonable

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based on the garden dimensions shown in the AR model or continue to estimate the total fencing cost as requested in the problem. Similar patterns were found in many student responses, where answers often ended with the final numerical result without further explanation. Classroom observations also showed that students rarely returned to the AR model after completing their calculations to review or verify their answers. These findings suggest that validation and interpretation were not yet a regular part of students' mathematical modeling process. This is reflected in the medium N-Gain scores for Validation ($g = 0.45$) and Interpretation ($g = 0.50$) presented in Table 8. This behavior indicates that the validation and interpretation stages of the modeling cycle did not occur naturally, even though students successfully completed the formulation and calculation stages.

The Effectiveness, Learning Contributions, and Limitations of AR-Supported Mathematical Modeling

The findings indicate that students' mathematical modeling skills improved significantly following the AR-supported learning intervention. The significant difference between pretest and posttest scores ($t(31) = 8.494$, $p < 0.001$) confirms that the intervention was effective, with the average N-Gain of 0.57 falling into the medium category. These results are consistent with previous studies reporting that AR integration in geometry instruction enhances students' spatial visualization and understanding (Del Cerro Velázquez & Méndez, 2021; Garzón & Acevedo, 2019). However, this study extends those findings by explicitly linking AR affordances to each stage of the mathematical modeling cycle, an aspect that has received limited empirical attention in prior research.

The qualitative findings help explain how AR facilitate students learning. Students reported that AR helped them visualize circular structures such as domes, windows, and arches more clearly than textbook images. By manipulating AR models, students could identify geometric elements such as radius, diameter, and central angle that were previously difficult to visualize from static diagrams (Lavicza et al., 2023; Miner & Alipour, 2024; Tarng et al., 2024). This supports constructivist learning theory, which emphasizes that students construct knowledge more effectively through meaningful, hands-on experiences (Geiger, 2020; Ziatdinov & Valles, 2022). These findings are also consistent with Greefrath (2011), who argued that digital tools can support students across the modeling stages, particularly in understanding the problem and constructing representations.

The use of historical building contexts also played an important role in supporting problem identification. When students encountered mathematical problems embedded in familiar local heritage sites, they were more engaged and could identify known and asked elements. This finding extends previous research by Cahyono (2018) and Cahyono & Ludwig (2019), who advocated for using historical environments and local heritage sites as contexts for mathematical modeling. The novelty of this study lies in applying AR specifically to historical building contexts in Semarang City, providing authentic and culturally relevant learning experiences that are often absent in conventional geometry instruction.

However, this study also revealed an important gap in students' modeling competence. Most students successfully applied circle formulas and performed calculations, but their written answers did not include any interpretation or validation of the results in the building context (Payadnya et al., 2023; Abakah, 2023, 2023; Siregar, 2025). This finding is consistent with the modeling cycle theory, which identifies validation and interpretation as the most challenging stages for students because they require moving back from the mathematical domain to the real-world context (Blum & Leiss, 2007; Stillman, 2019). While AR helped students visualize

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the real-world context, it did not automatically support them in comparing mathematical outputs with physical referents. This suggests that explicit scaffolding for the validation and interpretation stages is necessary, an aspect often overlooked in AR integration studies (Greefrath & Siller, 2024). A theoretical explanation for this pattern lies in cognitive demands at each modeling stage. AR provides visual and spatial support that addresses the perceptual challenges of problem identification and representation students can rotate, zoom, and inspect three-dimensional building structures in ways that static diagrams cannot. However, validation and interpretation require metacognitive and reflective reasoning: students must evaluate whether their numerical result is plausible in the real-world context, recognize inconsistencies with physical constraints, and articulate what the result means in practical terms (Schukajlow et al., 2018; Stillman, 2019). These higher-order processes are not triggered by visual affordances alone. Explicit scaffolding is therefore needed, such as guided reflection questions (e.g., 'Does this answer make sense for the actual building?'), estimation tasks connecting outputs to physical scale, and structured prompts requiring written contextual interpretation before an answer is accepted as complete (Greefrath & Siller, 2024; Krawitz et al., 2025).

It is important to position AR as a pedagogical tool rather than a complete instructional solution. This study does not claim that AR alone develops mathematical modeling competence. The evidence suggests AR is most effective as a visualization and contextualization scaffold for the early and middle modeling stages problem identification, geometric representation, and model formulation as reflected in the high N-Gain for Problem Identification ($g = 0.94$) and Model Formulation ($g = 0.73$). Conversely, the low gains in Representation ($g = 0.25$) and Interpretation ($g = 0.50$) indicate that AR without explicit instructional guidance for these stages is insufficient. A productive design should combine AR with structured modeling scaffolds: guided questioning, reflection protocols, and explicit modeling stage prompts. Future research should explore how such combined designs affect the full modeling cycle, especially stages that AR does not naturally support.

This study has several limitations. The sample consisted of only one class from a single school, limiting generalizability. The intervention was relatively short, consisting of only four meetings. The one-group pretest-posttest design without a control group does not allow for direct comparison with conventional instruction. Qualitative data were collected through informal questioning rather than in-depth interviews. Despite these limitations, the findings provide meaningful initial evidence on how AR integrated with local cultural heritage can support geometry-based mathematical modeling and offer practical directions for future research with larger samples, control groups, and explicit scaffolding strategies for the later modeling stages.

Conclusion

This study examined changes in students' mathematical modeling skills after they participated in an AR-supported geometry learning intervention based on Semarang City's historical buildings. The findings showed a statistically significant increase in students' scores from the pretest to the posttest, with an average N-Gain of 0.5755, which was categorized as medium improvement. The qualitative findings indicated that AR was particularly useful in supporting the early stages of the mathematical modeling cycle, especially visualization, problem identification, and mathematization. However, students continued to experience difficulties in the validation and interpretation stages. Many students were able to complete the required calculations but did not verify the reasonableness of their answers or relate the results back to the original building context. This pattern helps explain why the overall improvement remained

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in the medium rather than the high category. Theoretically, the findings support the view that AR can serve as a meaningful technological scaffold within the mathematical modeling cycle when its features are deliberately aligned with the cognitive demands of specific modeling stages. Practically, teachers should not rely on AR alone but should complement it with explicit scaffolding strategies, such as guided reflection questions, validation checklists, estimation activities, and structured prompts for contextual interpretation. The use of local historical buildings as learning contexts also appeared to increase students' engagement and provide culturally relevant connections between geometry and their surrounding environment.

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